2018 Fall EECS205003 Linear Algebra - Midterm II

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1. (12%) Suppose *A* =

2 4 3 2 5 7 4 9 10

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, please answer the following questions:

(a) (4%) Show whether the columns of *A* are linear independent or not.

(b) (4%) Find a basis of the orthogonal subspace of *C*(*A*).

(c) (4%) Find a basis of the orthogonal subspace of *C*(*AT*).

2. (17%) Please answer the following questions:

(a) (4%) Find the projection p of the vector b = (1*,* 2*,* 6) onto the plane *x* + *y* + *z* = 0 in R3. (b) (4%) Find the line which best fits the four data points (1*,* 2)*,*(2*,* 1)*,*(3*,* 3) and (4*,* 2) in the sense of least squares.

(c) (4%) Let *S* = *{*v1*,* v2*}* be the set of the following vectors in R4:

v1 =

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101 0

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*,* v2 =

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011 0

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*.*

Find the orthogonal basis of the subspace span(*S*) of R4. 

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(d) (5%) Use Gram Schmidt process to find the QR factorization of *A* =

1 2 1 2 0 3

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3. (16%) Let u1 =

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111 1

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*,* u2 =

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1*−*11 *−*1

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*,* u3 =

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*−*111 *−*1

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*,* u4 =

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*−*1

*−*1

1

1

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*,* y =



123 4

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*U*4*×*4 =u1 u2 u3 u4*.*

(a) (4%) Is *U*4*×*4 an orthogonal matrix? Please give your reason. (b) (4%) Find *U−*1.

(c) (4%) Suppose that y = *c*1u1 + *c*2u2 + *c*3u3 + *c*4u4. Find *c*3 and *c*4. (d) (4%) Let *W* = Span*{*u1*,* u2*,* u3*}*. Find the vector in *W* that is closest to y.

4. (13%) Consider the 4 by 4 matrices:

*A* =

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2*a c b* 0

*b a* + *d* 0 *b c* 0 *a* + *d c* 0 *c b* 2*d*

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1 101 201 301

2 102 202 302 3 103 203 303 4 104 204 304

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(a) (5%) Find determinant of *A*. (In terms of *a* + *d* and *ad − bc*).

(b) (4%) In this question, please find the relation between *a, b, c, d*, such that *A* has no inverse. (c) (4%) Find determinant of *AB*.

Please turn over to continue.

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5. (10%) Let *M* =

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*a b* 0 0

0 0 *a b c d* 0 0 0 0 *c d*

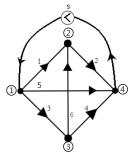
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, where *a > b >* 0 and *d > c >* 0,

(a) (5%) Find the determinant of the matrix *M*

(b) (5%) Find *M−*1

6. (17%) The following figure shows electrical network G. The conductance (*c*1*, c*2*, c*3*, c*4*, c*5*, c*6) = (1*,*12*,* 1*,*12*,* 1*,*14) to corresponding edges. And the current source *s* flows into node 1 and flows out of node 4. Please answer the following questions:



(a) (7%) Represent the incident matrix *A* and try to use loops in G to indicate a basis of the left nullspace of *A* instead of computing elimination.

(b) (10%) Suppose that potential at node 1 is *v*. Find potential at each node and current on each edge.

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